# LUNAR AIR TIDE IN THE CARIBBEAN AND ITS MONTHLY VARIATION

B. HAURWITZ* AND ANN D. COWLEY

National Center for Atmospheric Research, Boulder, Colo.

## ABSTRACT

The lunar air tide and the solar 24-, 12-, 8-, and 6-hourly oscillations have been determined for Willemstad, N.W.I. and Trinidad, B.W.I. Monthly means of these oscillations have been computed for Puerto Rico.

## 1. INTRODUCTION

The present paper is a further report on a continuing study to increase our knowledge of the worldwide distribution of the lunar atmospheric tide. It is a sequel to an earlier paper [1] in which determinations of the lunar semidiurnal tide, and incidentally of the solar-day oscillations, in the barometric pressure were reported for six stations in North and Central America. For Central America only one previous determination exists, by Chapman and Hardman [2] for Mexico City. In our previous paper we gave, among other data, the results of our lunar-tidal determinations for Balboa, Panama, and for San Juan and Aguadilla (Ramey AFB), both in Puerto Rico. The tides for these three stations as well as for Mexico City were determined for the annual mean and for the three seasons, commonly used, namely

D: November, December, January, February

J: May, June, July, August

E: March, April, September, October (Equinoctial)

We have now also computed the lunar air tide for Trinidad, B.W.I. and Willemstad, N.W.I. which will be given below, together with the solar-day oscillations.

The annual variation of the lunar tide is only in a very summary fashion represented by the mean values for the three seasons. The reason for this summary representation is, of course, that the radius of the probable-error circle of the determination is inversely proportional to the square root of the number of available data. Consequently, much longer series of data are required for a satisfactory determination of monthly values of $L_2$ than for annual or seasonal means, and so far, monthly means of $L_2$ have been determined for only ten stations or combinations of stations [3]. The stations of San Juan and Aguadilla in Puerto Rico are very close, about 100 km. apart, so that $L_2$ should be very similar at both stations. It appears therefore possible to combine both stations in order to have in effect a longer series permitting the determination of monthly mean values for the combined Puerto Rican stations. In our previous paper [1] we pointed out that there is a phase difference of about 7° between the annual values at both stations which cannot be explained by a change in the observation routine. In order to see whether this difference is statistically significant or only due to the limited data sample a formula given by Bartels [4] may be used. If $d$ be the distance between two points representing two oscillations in a harmonic dial (polar diagram), $e^2$ the sum of the squared radii of the two probable-error circles, then $2^{-d^2/e^2}$ is the probability that such a separation between the oscillations is due to pure chance. This probability is for the annual values 1:9, for the D months 1:2, for the J months 1:13, and for the E months 1:3. Thus only for the summer months could the difference between the two stations possibly be considered significant. It thus seems justified to combine the two stations in order to obtain a more detailed representation of the annual change of $L_2$. This is done in section 4.

The data for the two new stations, Trinidad and Willemstad, have been obtained from the National Weather Records Center in Asheville, N.C. Sea level pressures had to be used. Since both stations are almost at sea level (see table 1) the effect of the reduction to sea level on the determination of the oscillations to sea level is negligible. The procedures followed in the calculations are the same as described in our earlier paper, including the elimination of the days with bi-hourly

*Also affiliated with the University of Texas as University Professor (Atmospheric Science).

TABLE 1.—*List of stations*

| Station | Lat. °N. | Long. °W. | Elevation (ft.) | Years of record | Number of days |
|---|---|---|---|---|---|
| Willemstad, Curaçao, N.W.I. | | | | | |
| Hato Field | 12.2 | 68.9 | 30 | Sept. 1942–Dec. 1945 | 1152 |
| Plesman Airport | 12.2 | 69.0 | 0 | Jan. 1958–Mar. 1964 | 2273 |
| Trinidad, B.W.I. | | | | | |
| Waller A.F.B. | 10.6 | 61.2 | 14 | July 1941–Dec. 1948 | 2209 |
| Waller N.S. | 10.7 | 61.6 | 42 | Nov. 1947–Aug. 1955 | 2539 |
| Puerto Rico | | | | | |
| San Juan | 18.5 | 66.1 | 60–80 | Mar. 1945–Apr. 1962 | 6142 |
| Aguadilla | 18.5 | 67.1 | 220 | Jan. 1941–June 1958 | 6217 |

pressure changes greater than 4 mb. All computations were performed at NCAR on a CDC 3600 computer.

At both stations the observations were shifted from one location to another during the observation period. The relevant information is given in table 1, where the two Puerto Rican stations are included from our previous paper [1].

## 2. THE LUNAR TIDE

The results of the lunar tidal determination at Trinidad and Willemstad are shown in table 2. The oscillation is given in the usual form,

$$L_2(p_0)=l_2 \sin (30t+\lambda_2)$$

where $l_2$ is the amplitude given in microbars ($1\mu$b.$=10^{-3}$mb. $=1$ dyne cm.$^{-2}$), $t$ the local mean lunar time, $\lambda_2$ the phase constant in degrees. The radius of the probable error circle, $r_2$, is also given in microbars.

The phase angles of the annual means of $L_2$ at both Willemstad and Trinidad are larger than at the other stations in the Caribbean indicating that the maximum occurs here earlier than at Balboa or Puerto Rico. In particular, at Trinidad the pressure maximum occurs at the lunar transit. However, it is doubtful if the phase angle at Trinidad has been determined with sufficient accuracy. If the annual value of $L_2$ is determined separately for the two series of data shown in table 1 the following results are obtained:

| | |
|---|---|
| 1941–1948 | $56.6\mu$b. $\sin(2t+98.2°)\pm 4.6$ |
| 1947–1955 | $51.7\mu$b. $\sin(2t+82.7°)\pm 4.4$ |

A similar phase difference (of 12°) has been found for the solar semidiurnal pressure oscillation. This difference in the phase constant for the two periods suggests that, presumably during the first period, an unknown change in the observing routine was made. Allowances have been made for the known changes in the observing routine, consisting in making the observation on the half-hour instead of the whole hour. No discrepancy was noted for the two periods at Willemstad. Here the annual values of $L_2$ are for

1942–1945 $57.6\mu$b. $\sin(2t+83.2°)\pm 5.9$

and for

1958–1964 $60.9\mu$b. $\sin(2t+82.8°)\pm 2.9$

The seasonal means of $L_2$ show the characteristic variation of the amplitude, namely the maximum during the J (summer) season, the minimum during the D season, and the value during the E season intermediate, but closer to the J value. The seasonal values of the phase constant show the characteristic low values (late maximum) during the D season. During the other seasons the lunar pressure maximum occurs at or even before lunar transit. To permit an easy comparison of $L_2(p_0)$ in the Caribbean the results of the present and the previous paper for this area are combined in figure 1 in harmonic-dial representations. The lengths of the arrows which are centered at the stations give the amplitudes of the annual means according to the scale at the bottom of the figure. The angle between the local meridian and the arrow is equal to the phase constant minus 90°. Thus an arrow coinciding with the meridian corresponds to a lunar pressure maximum at lunar transit, an arrow pointing east of north ($\lambda_2<90°$) indicates a pressure maximum after lunar transit, and an arrow pointing west of north ($\lambda_2>90°$) shows a maximum before lunar transit. The seasonal means are shown by lines from the station with the appropriate letter near the end point of the line, but to keep the figure simple the lower halves of these lines have been omitted. The two Puerto Rican stations have been combined.

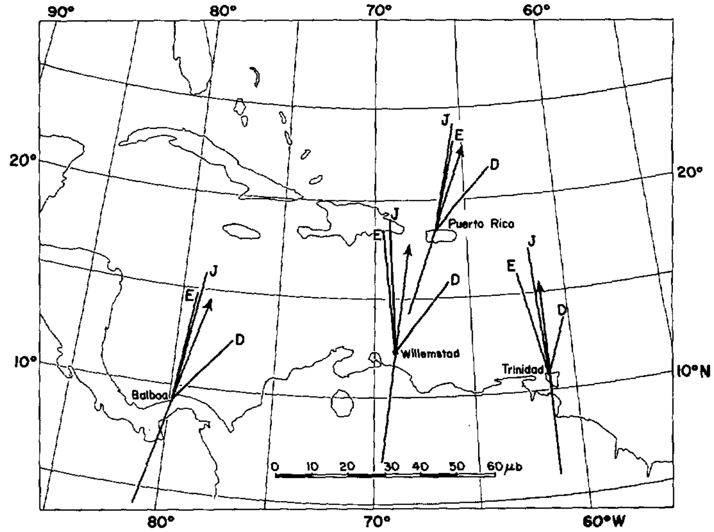


FIGURE 1.—The lunar barometric tide in the Caribbean. The lengths of the lines give the values of the amplitudes (only the upper half of each line is drawn for the seasonal means); the angles between the lines and the parallels of latitude correspond to the phase constants.

TABLE 2.—*Lunar semidiurnal pressure oscillation*

| Station | Season | $l_2$ ($\mu$b.) | $\lambda_2$ (deg.) | $r$ ($\mu$b.) |
|---|---|---|---|---|
| Willemstad, N.W.I. | Ann. | 59.8 | 82.9 | 2.9 |
| | D | 48.3 | 53.0 | 4.1 |
| | J | 73.0 | 91.8 | 6.4 |
| | E | 68.4 | 94.6 | 5.8 |
| Trinidad, B.W.I. | Ann. | 53.5 | 90.3 | 3.2 |
| | D | 34.2 | 69.8 | 4.3 |
| | J | 71.6 | 92.3 | 5.1 |
| | E | 59.8 | 100.5 | 5.2 |

## 3. THE SOLAR-DAY OSCILLATIONS

The solar-day oscillations for Willemstad and Trinidad which are obtained in the process of the determinations of $L_2$ are given in table 3 in the form

$$S_n=s_n \sin(15nt+\sigma_n)$$

where $t$ now denotes mean solar time. The radii of the

TABLE 3.—*Solar-day oscillations*

| Station | Season | $S_1$ ($\mu$b.) | $\sigma_1$ (deg.) | $r_1$ ($\mu$b.) | $S_2$ ($\mu$b.) | $\sigma_2$ (deg.) | $r_2$ | $S_3$ ($\mu$b.) | $\sigma_3$ (deg.) | $r_3$ ($\mu$b.) | $S_4$ ($\mu$b.) | $\sigma_4$ (deg.) | $r_4$ ($\mu$b.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Willemstad | Ann | 312.0 | 353.1 | ---- | 1251.4 | 160.7 | ---- | 12.8 | 133.5 | ---- | 51.4 | 207.9 | ---- |
| | D | 249.7 | 342.0 | 7.7 | 1297.9 | 165.9 | ---- | 150.7 | 67.8 | 3.7 | 68.2 | 215.1 | 3.0 |
| | J | 338.3 | 2.8 | 8.4 | 1149.5 | 155.2 | ---- | 54.5 | 201.0 | 4.2 | 23.0 | 181.1 | 4.0 |
| | E | 356.5 | 351.7 | 8.8 | 1311.0 | 160.5 | ---- | 25.3 | 160.0 | 3.7 | 64.1 | 209.1 | 4.2 |
| Trinidad | Ann | 566.4 | 342.9 | ---- | 1194.7 | 152.8 | ---- | 58.6 | 53.2 | ---- | 54.2 | 175.6 | ---- |
| | D | 547.4 | 332.5 | 6.9 | 1240.2 | 156.5 | ---- | 141.3 | 31.0 | 3.6 | 47.7 | 194.4 | 2.6 |
| | J | 559.6 | 356.9 | 7.2 | 1105.9 | 146.7 | ---- | 58.4 | 173.5 | 3.4 | 49.3 | 150.4 | 3.0 |
| | E | 616.8 | 340.8 | 8.7 | 1236.6 | 153.9 | ---- | 65.7 | 60.5 | 2.9 | 71.9 | 178.3 | 2.6 |

probable error circles are only given for the seasonal means, but not for the annual means, because the radius for the latter would also reflect the seasonal change. The values for $r_2$ are identical with those given for $L_2$ in table 2 after the effect of $L_2$ is allowed for, and have not been included in table 3.

$S_2$ has its minimum amplitude during the J season at both stations, in agreement with our earlier results [1] and with an analysis of the global distribution of the seasonal variations of $S_2$ [5]. For $S_3$ the maximum amplitude occurs during the D months as at the six stations analyzed in our earlier paper [1], and as found earlier by Hann [6] for the Northern Hemisphere in general. The oscillation $S_4$ has been discussed by Kertz [7] who showed that its main term, characterized by the Associated Legendre Function $P_5^4(\varphi)$ with a maximum around 25° latitude, has a maximum amplitude during the D season. Table 3 shows that this holds for Trinidad, although the maximum is not pronounced here, but not for Willemstad. Since both stations are well south of the maximum of $P_5^4(\varphi)$ this irregular behavior of the seasonal variation of $S_4$ is not surprising.

## 4. THE MONTHLY VARIATIONS FOR PUERTO RICO

The monthly mean values of $L_2$ for San Juan and Aguadilla are given in table 4. The values for every month have been combined to give a mean annual variation of $L_2$ for Puerto Rico. In combining these two stations both have been given equal weight since the length of the record is very nearly the same at both stations. The result is shown in figure 2 where the error circles, which have very nearly the same radius throughout the year, are only drawn for every second month. The curve for the monthly variation of $L_2$ agrees in general with similar curves for the other 10 stations for which monthly values of $L_2$ have been computed. Unique are the great phase differences between December and January and between February and March which are common to both Puerto Rican stations as table 4 shows.

The monthly values of the solar-day oscillations are given in table 5. Attention may be called here only to some well-known features. The minimum of $S_2$ occurs during the summer. The phase of $S_3$ is almost completely reversed between January and July. It is also interesting to note that the amplitude of $S_4$ conforms to the monthly variation found by Kertz for $P_5^4(\varphi)$, namely a maximum around the winter solstice.

TABLE 4.—*Monthly values of the lunar tide on Puerto Rico*

| Month | San Juan | | | Aguadilla | | |
|---|---|---|---|---|---|---|
| | $l_2$ ($\mu$b.) | $\lambda_2$ (deg.) | $r$ ($\mu$b.) | $l_2$ ($\mu$b.) | $\lambda_2$ (deg.) | $r$ ($\mu$b.) |
| January | 41 | 16 | 9 | 46 | 19 | 8 |
| February | 41 | 42 | 9 | 41 | 29 | 11 |
| March | 41 | 86 | 9 | 55 | 91 | 9 |
| April | 69 | 92 | 10 | 51 | 76 | 9 |
| May | 59 | 96 | 8 | 49 | 70 | 11 |
| June | 63 | 83 | 7 | 62 | 83 | 8 |
| July | 76 | 85 | 7 | 56 | 74 | 9 |
| August | 55 | 82 | 8 | 52 | 76 | 10 |
| September | 61 | 72 | 8 | 34 | 49 | 12 |
| October | 48 | 85 | 8 | 50 | 74 | 10 |
| November | 52 | 75 | 8 | 61 | 71 | 8 |
| December | 60 | 69 | 7 | 60 | 62 | 8 |

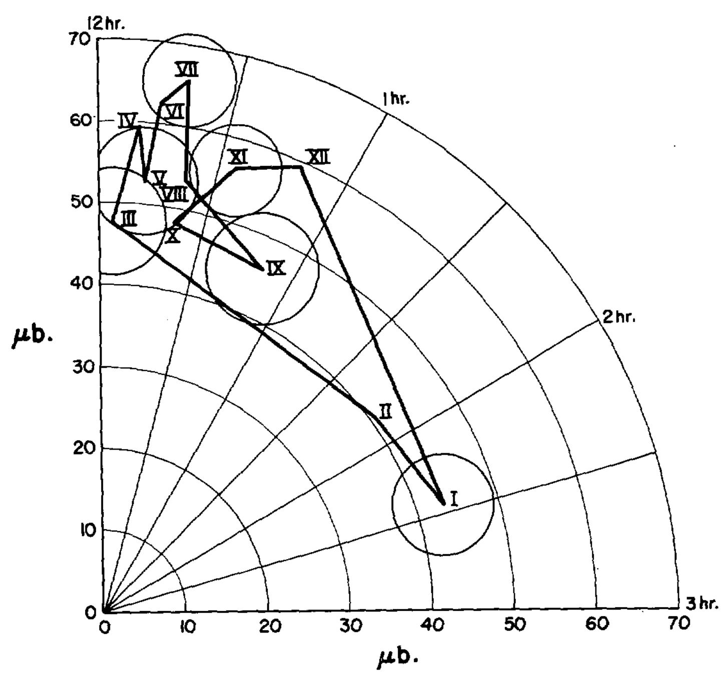


FIGURE 2.—Harmonic dial of the monthly means of the lunar barometric tide in Puerto Rico. Roman numerals give the months. Only every second probable-error circle is plotted.

TABLE 5.—*Monthly values of solar-day oscillations on Puerto Rico*

| Station | Month | $S_1$ ($\mu$b.) | $\sigma_1$ (deg.) | $r_1$ ($\mu$b.) | $S_2$ ($\mu$b.) | $\sigma_2$ (deg.) | $S_3$ ($\mu$b.) | $\sigma_3$ (deg.) | $r_3$ ($\mu$b.) | $S_4$ ($\mu$b.) | $\sigma_4$ (deg.) | $r_4$ ($\mu$b.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| San Juan | January | 272 | 334 | 12 | 1153 | 154 | 206 | 11 | 6 | 80 | 212 | 5 |
| | February | 323 | 332 | 15 | 1166 | 149 | 147 | 14 | 6 | 79 | 163 | 5 |
| | March | 339 | 342 | 15 | 1218 | 149 | 54 | 55 | 5 | 50 | 140 | 5 |
| | April | 367 | 347 | 15 | 1118 | 152 | 56 | 142 | 7 | 50 | 119 | 7 |
| | May | 293 | 12 | 14 | 985 | 152 | 92 | 165 | 7 | 38 | 132 | 6 |
| | June | 263 | 21 | 13 | 863 | 149 | 113 | 166 | 6 | 22 | 123 | 5 |
| | July | 170 | 8 | 11 | 840 | 145 | 119 | 172 | 6 | 41 | 138 | 5 |
| | August | 220 | 7 | 15 | 914 | 146 | 76 | 160 | 6 | 52 | 162 | 6 |
| | September | 282 | 359 | 14 | 1012 | 150 | 64 | 73 | 6 | 61 | 176 | 7 |
| | October | 325 | 5 | 13 | 1101 | 158 | 101 | 37 | 6 | 70 | 189 | 5 |
| | November | 314 | 354 | 12 | 1097 | 159 | 145 | 16 | 5 | 62 | 198 | 5 |
| | December | 312 | 350 | 13 | 1110 | 157 | 182 | 15 | 6 | 70 | 210 | 5 |
| Aguadilla | January | 296 | 329 | 14 | 1192 | 148 | 220 | 357 | 4 | 65 | 186 | 4 |
| | February | 328 | 319 | 15 | 1180 | 143 | 173 | 3 | 5 | 56 | 142 | 5 |
| | March | 360 | 320 | 13 | 1190 | 143 | 88 | 33 | 7 | 51 | 110 | 6 |
| | April | 359 | 324 | 12 | 1085 | 146 | 67 | 79 | 6 | 42 | 107 | 4 |
| | May | 214 | 340 | 14 | 954 | 143 | 68 | 125 | 5 | 27 | 118 | 4 |
| | June | 215 | 345 | 16 | 840 | 141 | 94 | 136 | 6 | 30 | 123 | 5 |
| | July | 199 | 349 | 15 | 829 | 139 | 88 | 138 | 6 | 37 | 123 | 4 |
| | August | 202 | 349 | 15 | 890 | 140 | 53 | 109 | 6 | 50 | 152 | 5 |
| | September | 247 | 338 | 13 | 997 | 143 | 73 | 54 | 6 | 59 | 161 | 5 |
| | October | 277 | 348 | 14 | 1105 | 151 | 120 | 24 | 6 | 71 | 173 | 5 |
| | November | 310 | 342 | 12 | 1153 | 152 | 168 | 4 | 5 | 69 | 176 | 5 |
| | December | 328 | 344 | 16 | 1164 | 151 | 199 | 2 | 6 | 63 | 190 | 5 |


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